

Color superconductivity

Igor A. Shovkovy



Frankfurt Institute for Advanced Studies
Johann W. Goethe-Universität
Max-von-Laue-Str. 1
60438 Frankfurt am Main, Germany

Outline

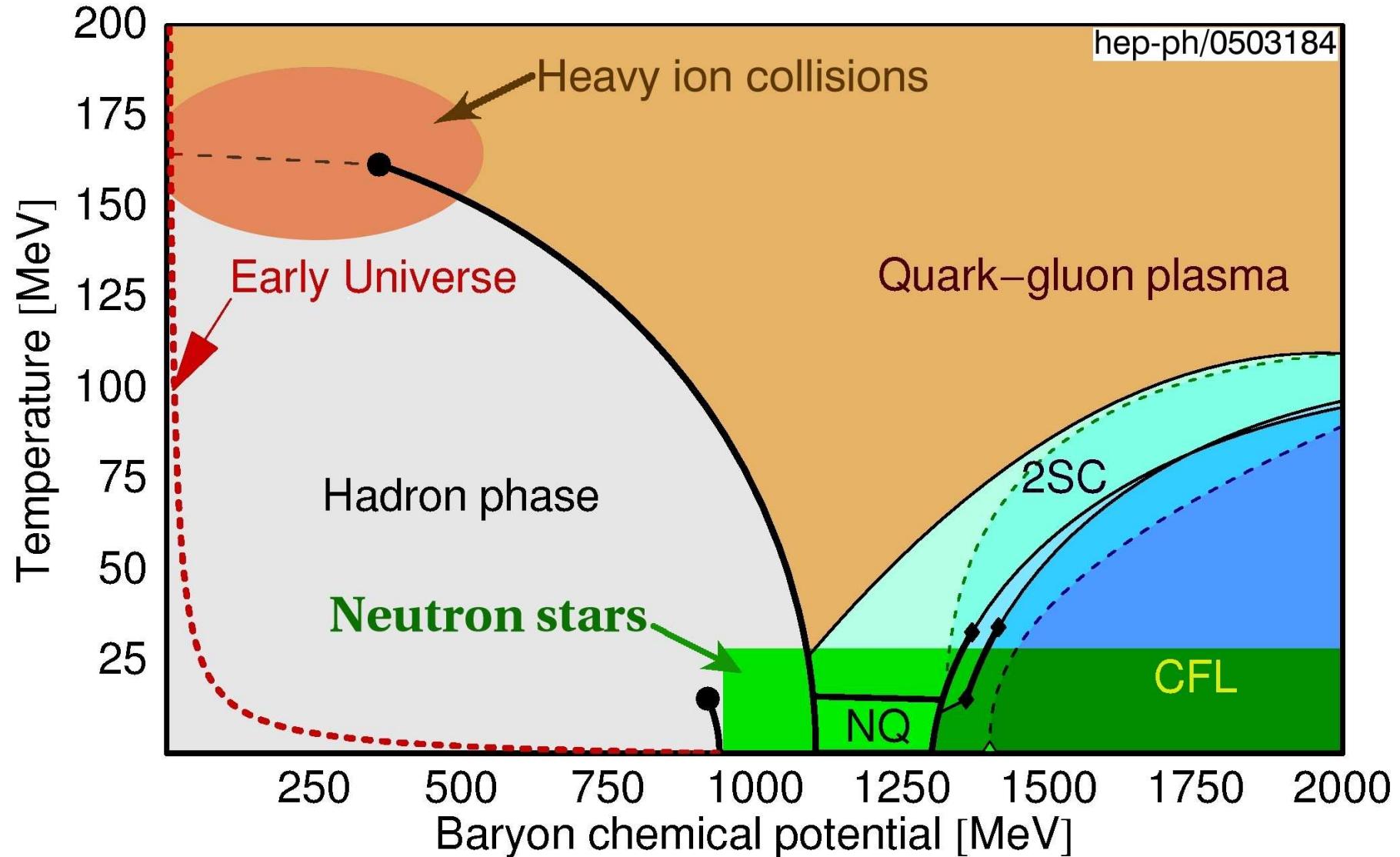
I. Introduction into color superconductivity

- Cold dense baryonic matter
- Color superconductivity
 - 2SC (2 quark flavors)
 - CFL (3 quark flavors)
 - Spin-1 color superconductivity (1 quark flavor)

II. Cooper pairing under stress

- Neutrality vs. color superconductivity
- Unconventional pairing in color superconductors
- Gapless, crystalline, gluonic, p-waves and all that . . .
- Current status, future directions, etc.
- Summary

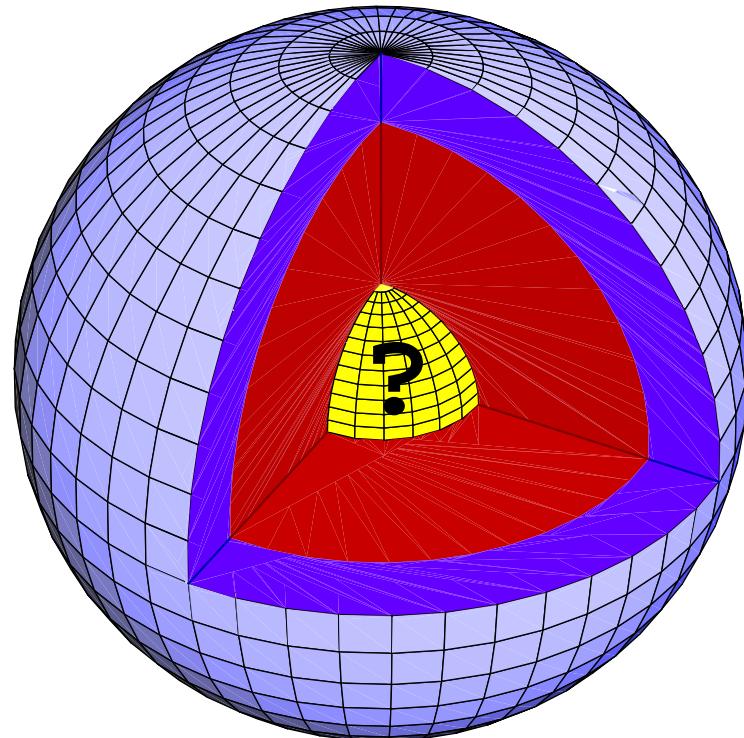
Conjectured phase diagram of QCD



Dense baryonic matter in Nature

Compact (neutron) stars

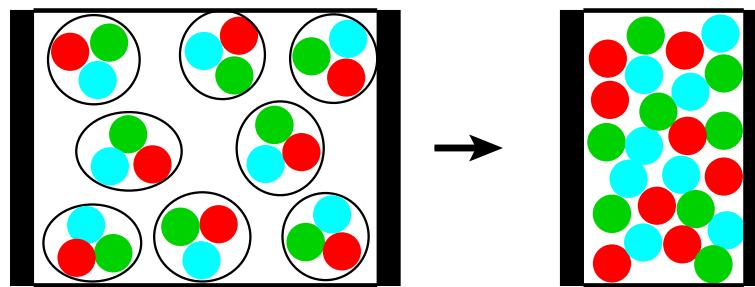
- Radius:
 $R \simeq 10 \text{ km}$
- Mass:
 $1.25M_{\odot} \lesssim M \lesssim 2M_{\odot}$
- Core temperature:
 $10 \text{ keV} \lesssim T \lesssim 10 \text{ MeV}$
- Surface magnetic field:
 $10^8 \text{ G} \lesssim B \lesssim 10^{14} \text{ G}$
(→ talk by Manuel)
- Rotational period:
 $1.6 \text{ ms} \lesssim P \lesssim 12 \text{ s}$



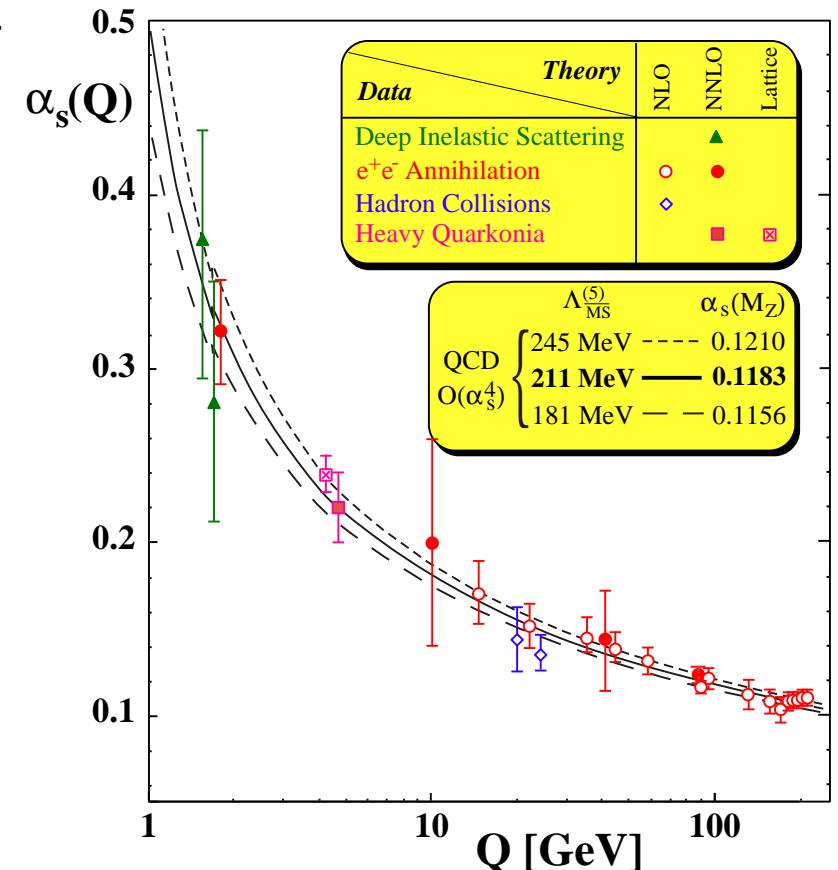
Central densities in stars should be rather high: $\rho_c \gtrsim 5\rho_0$

Very dense baryonic matter

Baryons at high density \rightarrow quark matter



- Asymptotic freedom: $\alpha_s(\mu) \ll 1$
 $\mu \gg \Lambda_{QCD}$ [Gross&Wilczek; Politzer, '73]
- \Rightarrow weakly interacting regime (?)
[Collins&Perry, '75]



:(Unfortunately, realistic densities in stars may not be sufficiently large:

$$\rho \lesssim 10\rho_0, \text{ where } \rho_0 \approx 0.15 \text{ fm}^{-3} \quad \Rightarrow \quad \mu \lesssim 500 \text{ MeV} \quad \Rightarrow \quad \alpha_s \simeq 1$$

Two complimentary approaches

(i) QCD (from first principles):

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_f^\alpha (i\gamma^\mu \partial_\mu + \gamma^0 \mu_f + g T_{\alpha\beta}^a \gamma^\mu A_\mu^a - m_f) \psi_f^\beta - \frac{1}{4} F_{\mu\nu}^a F^{a,\mu\nu}$$

- predictions are reliable when $\mu \gg \Lambda_{QCD}$

(ii) Phenomenological (e.g., NJL-type) models fitted to reproduce basic properties of vacuum QCD and/or nuclear matter, e.g.,

$$\mathcal{L}_{\text{NJL}} = \bar{\psi}_f^\alpha (i\gamma^\mu \partial_\mu + \gamma^0 \mu_f - m_f) \psi_f^\beta + \frac{g^2}{2} (\bar{\psi} \gamma^\mu T^a \psi) (\bar{\psi} \gamma_\mu T^a \psi)$$

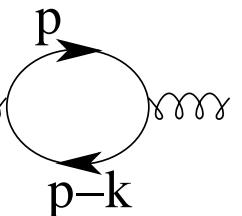
- with parameters fitted to reproduce QCD properties at $\rho \lesssim \rho_0$



Note: densities of interest: $3\rho_0 \lesssim \rho \lesssim 10\rho_0$

QCD: one-gluon exchange (with screening)

Gluon propagator in dense medium:

$$\mathcal{D}_{\mu\nu}^{-1}(k) = D_{0,\mu\nu}^{-1}(k) - \Pi_{\mu\nu}(k), \quad \text{i.e.,} \quad \overset{\text{k}}{\text{---}} = \overset{\text{k}}{\text{---}} + \text{---} \quad \text{---} \quad \text{---}$$


Electrical Debye screening and magnetic dynamical screening
 [Son,hep-ph/9812287]:

$$i\mathcal{D}_{\mu\nu}(k_4, |\vec{k}|) \simeq -\frac{O_{\mu\nu}^{(el)}}{k_4^2 + |\vec{k}|^2 + 2M_D^2} - \frac{|\vec{k}| O_{\mu\nu}^{(mag)}}{|\vec{k}|^3 + \pi M_D^2 |k_4|/2},$$

where $M_D^2 = \alpha_s N_f \mu^2 / \pi$ is the Debye mass

Magnetic interaction is long-ranged in (near-)static limit, $k_4 \lesssim |\vec{k}| \ll \mu$

QCD: gap equation & solution

Gap equation:

$$\begin{array}{c} \text{---} \triangle \text{---} \\ p \end{array} = \begin{array}{c} \text{---} \bullet \text{---} \triangle \text{---} \bullet \text{---} \\ k \end{array} \Rightarrow \Delta_p \simeq \frac{\alpha_s}{9\pi} \int \frac{dk \Delta_k}{\sqrt{k^2 + \Delta_k^2}} \underbrace{\ln \frac{\lambda \mu}{|k - p|}}_{\text{"extra log"}}$$

Approximate solution for the gap

$$\Delta_0 \simeq \lambda \mu \underbrace{\exp \left(-\frac{3\pi^{3/2}}{2^{3/2} \sqrt{\alpha_s}} \right)}_{\text{long-range magnetic gluons}}$$

where

$$\lambda = \underbrace{\frac{2(4\pi)^{3/2}}{\alpha_s^{5/2}}}_{\text{electric gluons}} \times \underbrace{\exp \left(-\frac{4 + \pi^2}{8} \right)}_{\text{quark self-energy corrections}} \times \underbrace{(\text{other sub-sub-leading corrections})}_{\text{so far unknown}}$$

Many types of color superconductors

- Density of quark matter is controlled by chemical potential μ
- Quarks of mass m_i appear if $\mu_i > m_i$
- Different compositions \rightarrow different types of superconductors

1 quark flavor e.g., <i>up/down</i>	2 quark flavors <i>up and down</i>	3 quark flavors <i>up, down and strange</i>
CSL: 	2SC	CFL
Planar: 		
A/polar: 		
Meissner effect: ✓ [*] superfluidity: ✓ [*]	Meissner effect: ✗ superfluidity: ✗	Meissner effect: ✗ superfluidity: ✓

$N_f = 2$ color superconductivity (2SC)

Simplest case, 2SC phase [Barrois,'78; Bailin&Love,'84]

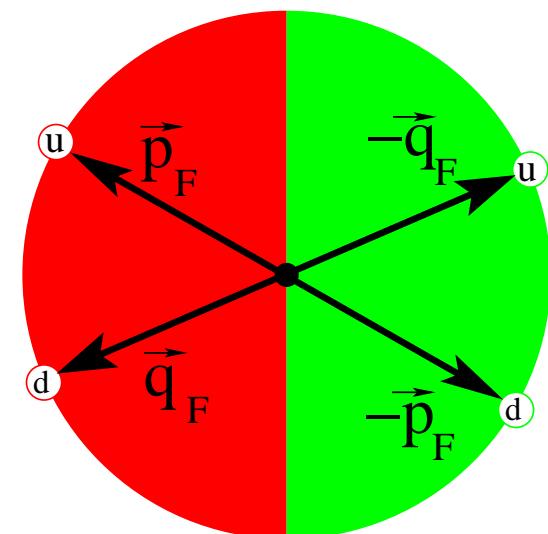
- $N_f = 2$: “up” and “down”
- $N_c = 3$: “red”, “green” and “blue”

$$\langle \mathbf{u}_p \mathbf{d}_{-p} \rangle = - \langle \mathbf{u}_q \mathbf{d}_{-q} \rangle \neq 0$$

Diquark condensate:

$$\phi_3 = \left\langle (\bar{\Psi}^C)_i^{\alpha} \varepsilon^{ij} \varepsilon_{\alpha\beta 3} C \gamma^5 \Psi_j^{\beta} \right\rangle \neq 0$$

(Pauli principle)



Note:

- Same colors and same flavors do not pair (spin-0 channel)
- Only two colors of quarks participate in Cooper pairing

Symmetries of 2SC state

- Diquark condensate:

$$\langle (\bar{\Psi}^C)_i^{\alpha} \gamma_5 \Psi_j^{\beta} \rangle \sim \varepsilon^{3\alpha\beta} \epsilon_{ij} \Delta$$

[Alford et al, hep-ph/9711395], [Rapp et al, hep-ph/9711396]

- ✓ baryon number $U(1)_B \rightarrow \tilde{U}(1)_B$ with $\tilde{B} = B - \frac{2}{\sqrt{3}}T_8$
(quark matter is not superfluid)
- ✓ gauge symmetry $U(1)_{\text{em}} \rightarrow \tilde{U}(1)_{\text{em}}$ with $\tilde{Q} = Q - \frac{1}{\sqrt{3}}T_8$
(there is no Meissner effect)
- chiral $SU(2)_L \times SU(2)_R$ — intact
- approximate axial $U(1)_A$ is broken $\rightarrow 1$ pseudo-NG boson
- color gauge symmetry $SU(3)_c \rightarrow SU(2)_c$ (Higgs mechanism)

$N_f = 3$ color superconductivity

Diquark condensate:

$$\left\langle (\bar{\psi}^C)_i^a \varepsilon^{ijk} \varepsilon_{abc} C \gamma^5 \psi_j^b \right\rangle \sim \delta_k^c$$

or, in terms of chiral fields,

$$\left\langle \psi_{L,i}^{a,\alpha} \varepsilon^{ijk} \varepsilon_{abc} \varepsilon_{\alpha\beta} \psi_{L,j}^{b,\beta} \right\rangle = - \left\langle \psi_{R,i}^{a,\dot{\alpha}} \varepsilon^{ijk} \varepsilon_{abc} \varepsilon_{\dot{\alpha}\dot{\beta}} \psi_{R,j}^{b,\dot{\beta}} \right\rangle \sim \delta_k^c$$

Color-flavor locking:

[Alford, Rajagopal, & Wilczek, hep-ph/9804403]

$\langle LL \rangle$	is invariant under	$g_{L,\text{flavor}} \otimes g_{\text{color}}$	if	$g_{\text{color}} \equiv g_{L,\text{flavor}}^{-1}$
$\langle RR \rangle$	is invariant under	$g_{R,\text{flavor}} \otimes g_{\text{color}}$	if	$g_{\text{color}} \equiv g_{R,\text{flavor}}^{-1}$

i.e., residual global symmetry is $SU(3)_{L+R}$

As in vacuum, there appear 8 corresponding Nambu-Goldstone bosons:

$$\pi^0, \pi^\pm, K^0, \bar{K}^0, K^\pm, \eta$$

Symmetries of CFL ground state

- ✓ chiral $SU(3)_L \times SU(3)_R$ is broken down to $SU(3)_{L+R+c}$
 $\rightarrow 8$ (pseudo-)NG bosons, i.e., $\pi^0, \pi^\pm, K^\pm, K^0, \bar{K}^0, \eta$
(almost like in vacuum QCD)
- ✓ baryon number $U(1)_B$ is broken $\rightarrow 1$ NG boson (ϕ)
(quark matter is superfluid)
- ✓ gauge symmetry $U(1)_{\text{em}} \rightarrow \tilde{U}(1)_{\text{em}}$ with $\tilde{Q} = Q + \frac{2}{\sqrt{3}}T_8$
(there is no Meissner effect)
- approximate axial $U(1)_A$ is broken $\rightarrow 1$ pseudo-NG boson (η')
- color gauge symmetry $SU(3)_c$ is broken by Anderson-Higgs mechanism
 $\rightarrow 8$ massive gluons

$N_f = 1$ color superconductivity

- Cooper pair: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes |\uparrow\uparrow\rangle_{J=1}$

- Diquark condensate:

$$\langle (\bar{\Psi}^C)^\alpha \gamma_5 \Psi^\beta \rangle \simeq \varepsilon^{\alpha\beta c} \Delta_{c\delta} \left(\hat{\mathbf{k}}^\delta \sin \theta + \gamma_\perp^\delta(\vec{\mathbf{k}}) \cos \theta \right)$$

[Iwasaki & Iwado, 1995], [Schäfer, hep-ph/0006034], [Alford et al, hep-ph/0210106]

- Many possibilities, e.g., see [Schmitt, nucl-th/0412033]:
 - Color-spin-locked phase: $\Delta_{c\delta} = \delta_{c\delta} \rightarrow$ largest pressure (?)
 - Planar phase: $\Delta_{c\delta} = \delta_{c\delta} - \delta_{c3}\delta_{\delta 3}$
 - Polar phase: $\Delta_{c\delta} = \delta_{c3}\delta_{\delta 3}$
 - A-phase: $\Delta_{c\delta} = \delta_{c3}(\delta_{\delta 1} + i\delta_{\delta 2}) \rightarrow$ unusual neutrino emission
- Many similarities with superfluidity in ${}^3\text{He}$. . .

Conditions inside stars: β -equilibrium

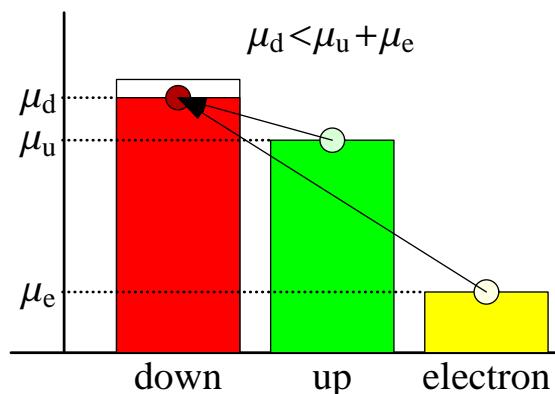
Weak processes



should have equal rates

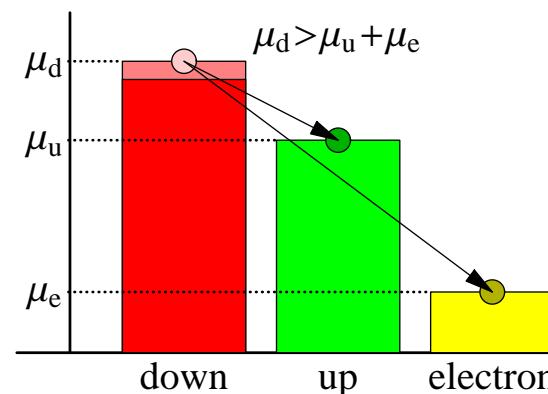
Too few d-quarks

Energy



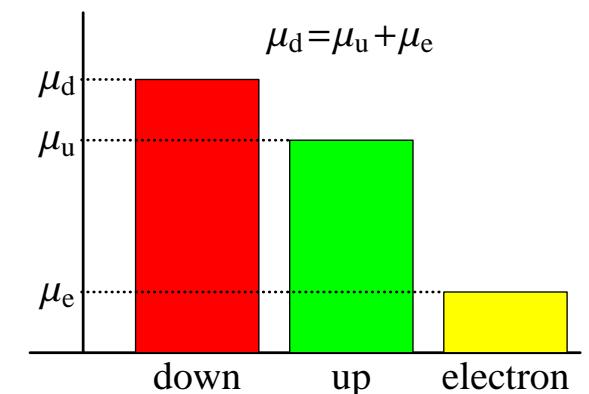
Too many d-quarks

Energy



β -equilibrium

Energy



$$\beta\text{-equilibrium} \Rightarrow \mu_d = \mu_u + \mu_e$$

Conditions inside stars: charge neutrality

Matter inside a star should be electrically neutral $n_Q = 0$
i.e.,

$$\underbrace{\frac{2}{3}n_u - \frac{1}{3}n_d - n_e = 0}_{\text{non-strange quark matter}} \quad \left(\text{or} \quad \underbrace{\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0}_{\text{strange quark matter}} \right)$$

If $n_Q \neq 0$, the Coulomb energy of a quark matter core of radius R is

$$E_{\text{Coulomb}} \sim n_Q^2 R^5 \sim M_\odot c^2 \left(\frac{n_Q}{10^{-15} e/\text{fm}^3} \right)^2 \left(\frac{R}{1 \text{ km}} \right)^5$$

e.g., $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/\text{fm}^3 \Rightarrow E_{\text{Coulomb}}^{2\text{SC}} \sim 10^{26} M_\odot c^2 \gg M_\odot c^2$ *

* Note: this is about 1000 000 000 000 000 000 000 000 times more powerful than a supernova explosion!

Unconventional Cooper pairing, $N_f = 2$

- The “best” Cooper pairing occurs when $\mu_d \approx \mu_u$, i.e.,

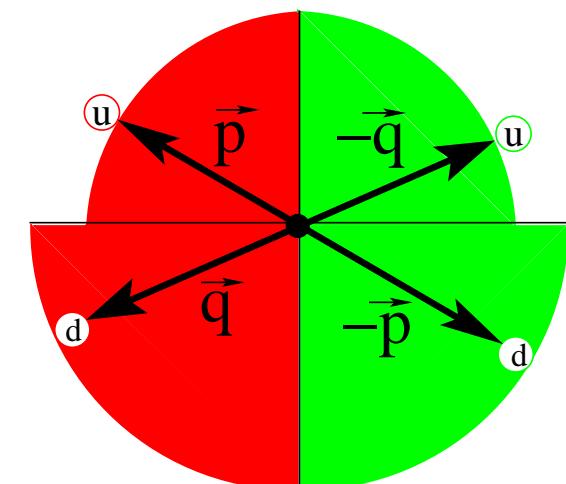
$$n_d \simeq \frac{\mu_d^3}{\pi^2} \simeq \frac{\mu_u^3}{\pi^2} \simeq n_u$$

- Neutral matter, however, appears when $n_d \approx 2n_u$
- There are not enough electrons in β equilibrium to help ...

$$\mu_e = \mu_d - \mu_u \approx 2^{1/3} \mu_u - \mu_u \approx \frac{1}{4} \mu_u \Rightarrow n_e \approx \frac{1}{192} n_u \ll n_u$$

Therefore, Cooper pairing is unavoidably distorted by the “mismatch”

$$\delta\mu \equiv \frac{\mu_d - \mu_u}{2} = \frac{\mu_e}{2} \neq 0$$



What happens then?

Gapless 2SC phase

Competition: $\delta\mu$ vs. Δ_0 (where Δ_0 is the gap at $\delta\mu = 0$)

The “winner” is determined by the diquark coupling strength

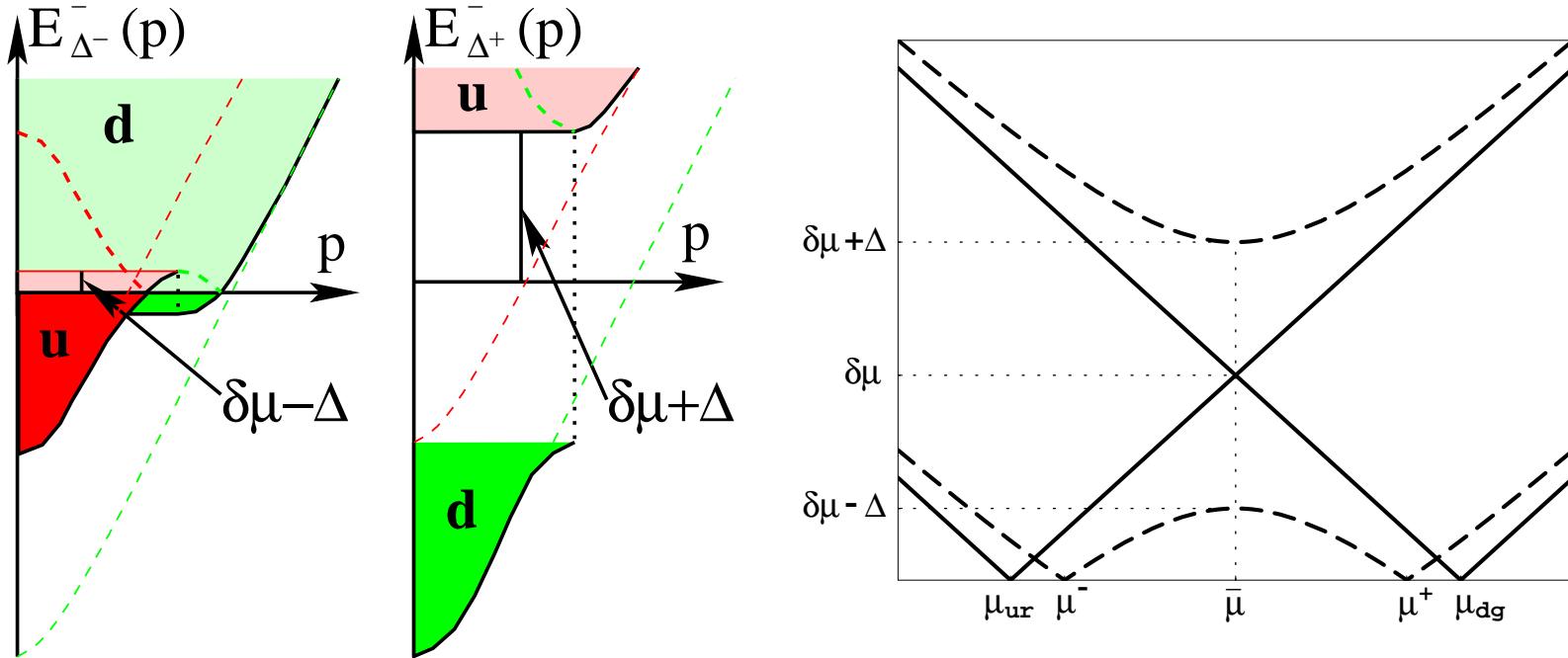
[Shovkovy&Huang, Phys. Lett. **B 564** (2003) 205]

1. $\delta\mu \gtrsim \Delta_0$ — the mismatch does not allow Cooper pairing:
normal phase is the ground state
2. $\delta\mu \lesssim \frac{1}{2}\Delta_0$ — coupling is strong enough to win over the mismatch:
2SC is the ground state
3. $\frac{1}{2}\Delta_0 \lesssim \delta\mu \lesssim \Delta_0$ — regime of intermediate coupling strength:
the ground state is the gapless 2SC phase

Quasiparticle spectrum in g2SC phase

“Intermediate” coupling

[Huang&Shovkovy, Nucl. Phys. **A 729** (2003) 835]



$$E_{\Delta\pm}(p) = \left| \sqrt{(p - \bar{\mu})^2 + \Delta^2} \pm \delta\mu \right|$$

i.e., the energy gaps in the quasiparticle spectra are

0

&

$\Delta + \delta\mu$

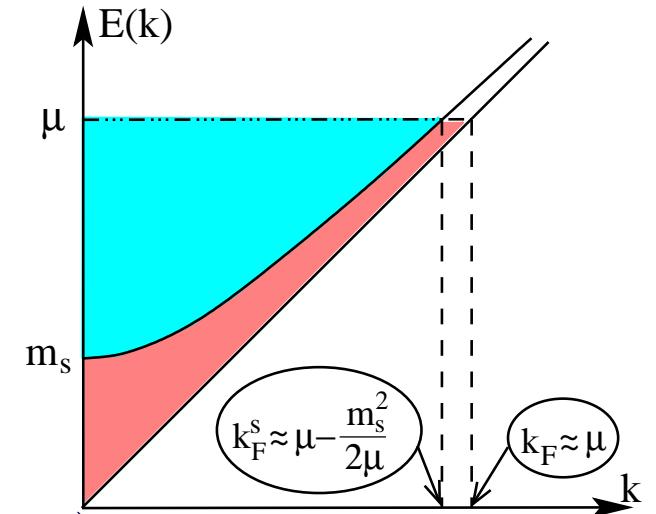
$N_f = 2 + 1$ color superconductivity, $0 < m_s < \infty$

Fermi momentum of strange quarks is lowered:

$$k_F^s \simeq \mu - \frac{m_s^2}{2\mu}$$

Then, the ground state is defined by:

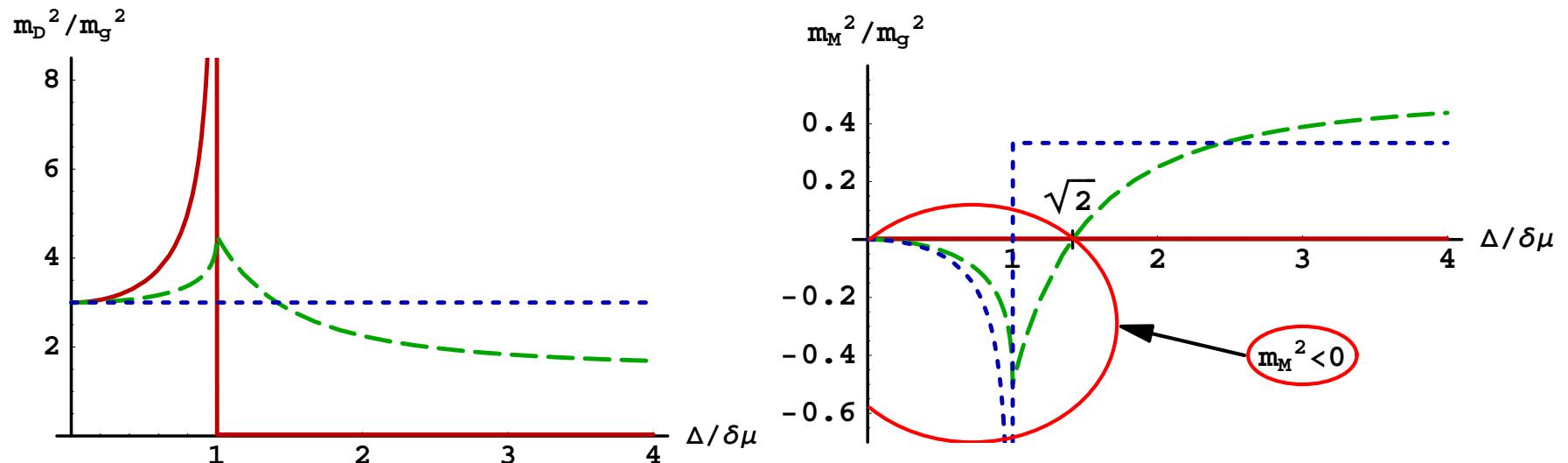
- (?) only condensates of same flavor (spin-1 channel)
- (?) only superconductivity of up and down quarks (2SC or g2SC)
- (?) gapless CFL phase (\oplus yet unknown stabilization mechanism)
[Alford, Kouvaris & Rajagopal, hep-ph/0311286]
- (?) crystalline pairing (nonzero momentum pairing, LOFF)
[Alford, Bowers & Rajagopal, hep-ph/0008208]
- (?) P-wave kaon condensates
[Schäfer, hep-ph/0508190]



Chromomagnetic instability

Abnormal results for gluon screening masses ($N_f = 2$):

[Huang & Shovkovy, Phys. Rev. D **70** (2004) 051501(R)]



$A = 1, 2, 3$ — red solid line

$A = 4, 5, 6, 7$ — green long-dash line

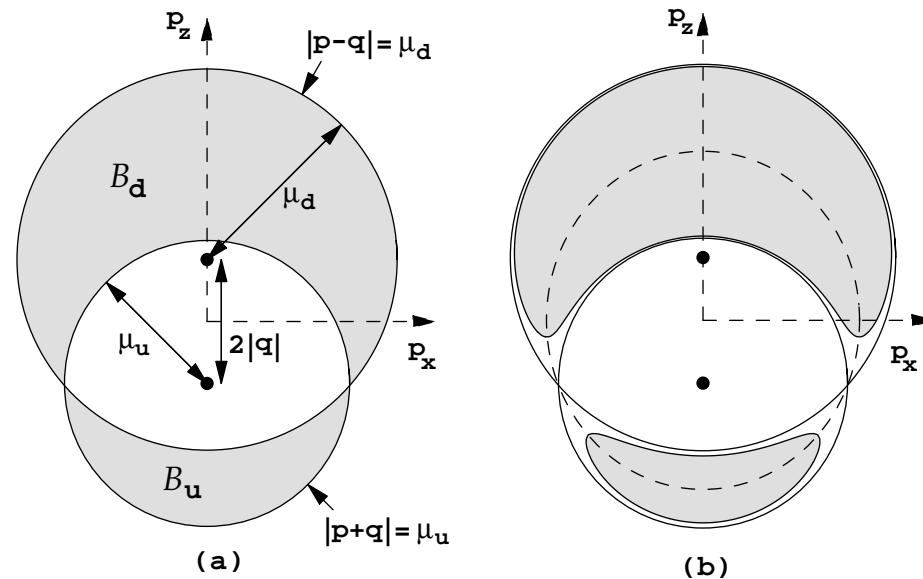
$A = \tilde{8}$ — blue short-dash line

$N_f = 3$: [Casalbuoni, et al, PLB**605** (2005) 362], [Fukushima, PRD**72** (2005) 074002]

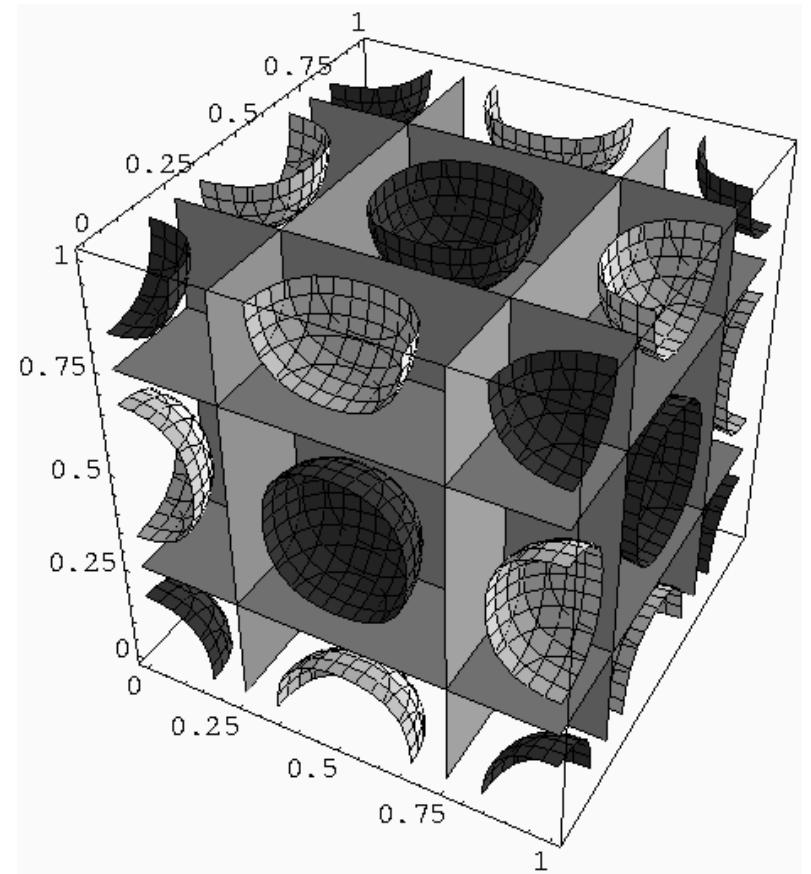
Crystalline phase (LOFF)

[Alford, Bowers & Rajagopal, hep-ph/0008208]

Cooper pairs with nonzero momenta:



(→ talk by Fukushima)



[Bowers, hep-ph/0305301]

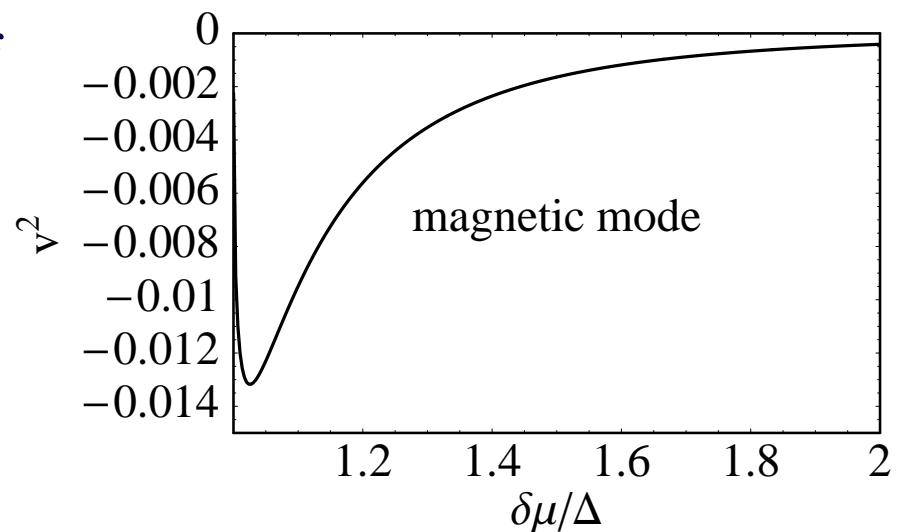
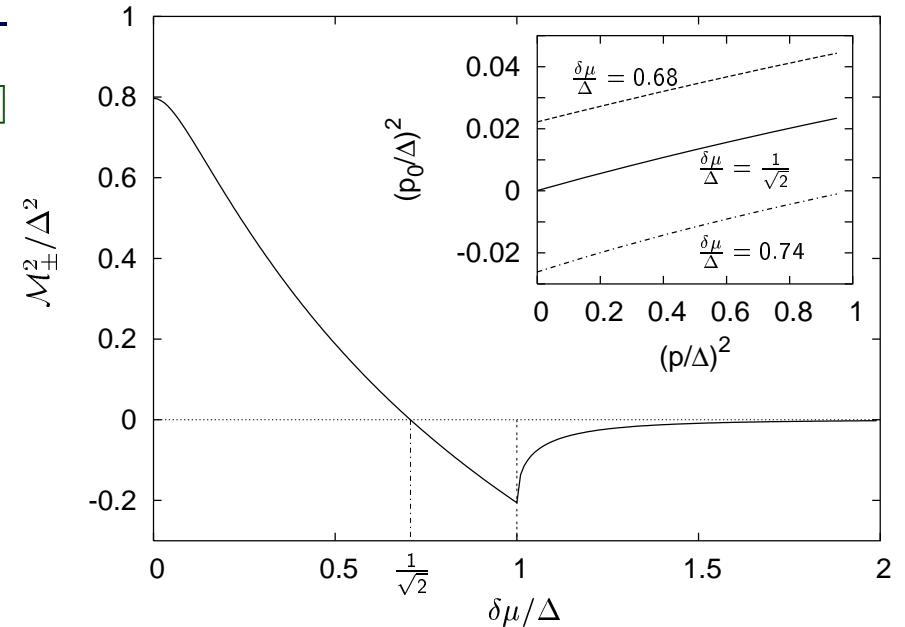
Origin of instability

Collective modes with quantum numbers of gluons [Gorbar et al, hep-ph/0602221]

- 4-7th: $p_0^2 = m^2 + v^2 p^2$
 $m^2 > 0$ for $\Delta > \sqrt{2}\delta\mu$
 $m^2 < 0$ for $\Delta < \sqrt{2}\delta\mu$
- 8th: $p_0^2 = v^2 p^2$ with $v^2 < 0$
 appearing only for $\Delta < \delta\mu$

Two types of tachyons \rightarrow two type of ground states

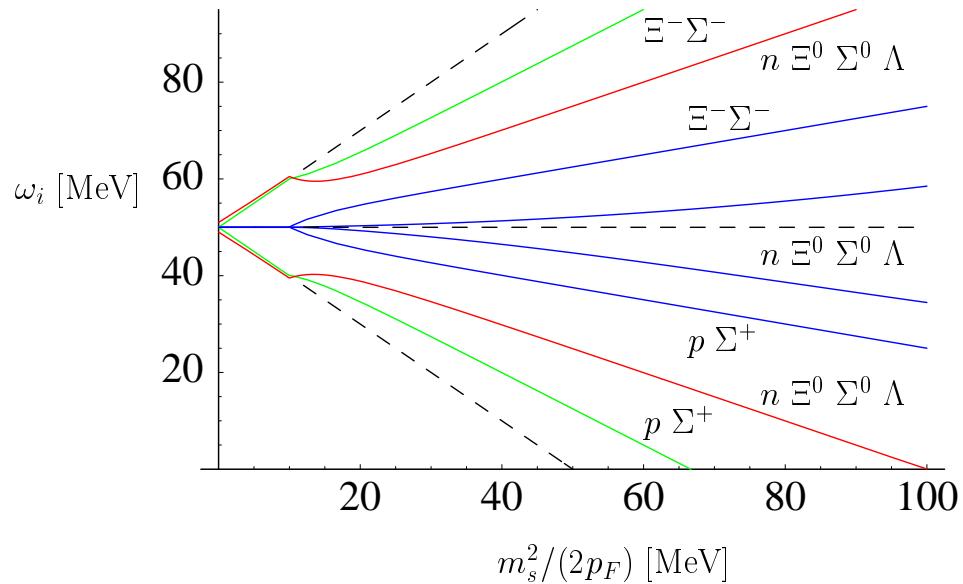
- 4-7th: $\langle A^{(4)} \rangle = \text{const}$
 [Gorbar et al, hep-ph/0507303]
- 8th: $\langle A^{(8)}(x) \rangle \neq \text{const}$
 i.e., more than 1-wave LOFF



P-wave kaon condensation

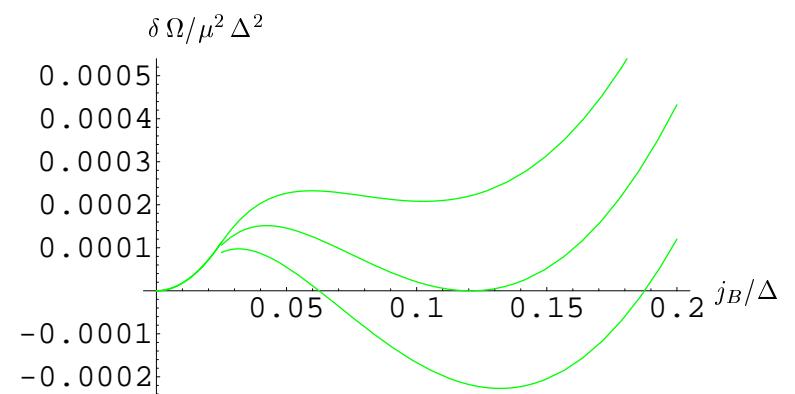
Baryon spectrum in CFL phase

[Kryjevski & Schäfer, hep-ph/0407329]



Within the framework of effective theory:

P-wave meson condensation



[Kryjevski, hep-ph/0508180],

[Schäfer, hep-ph/0508190]

See also

[Son & Stephanov, cond-mat/0507586] \Rightarrow



No instabilities

Observational data as a tool

Cooling of neutron stars:

(→ talk by Reddy)

(i) thermal relaxation: $t \lesssim 100$ yr

(ii) neutrino cooling: $100 \lesssim t \lesssim 10^6$ yr

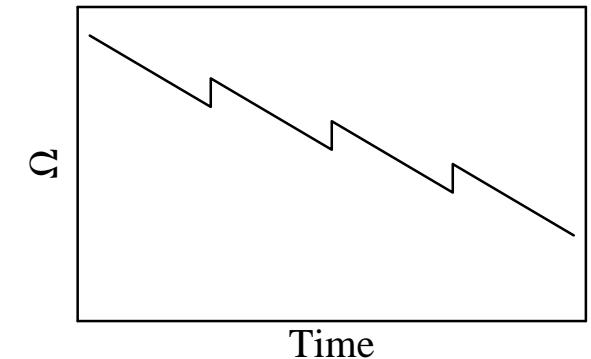
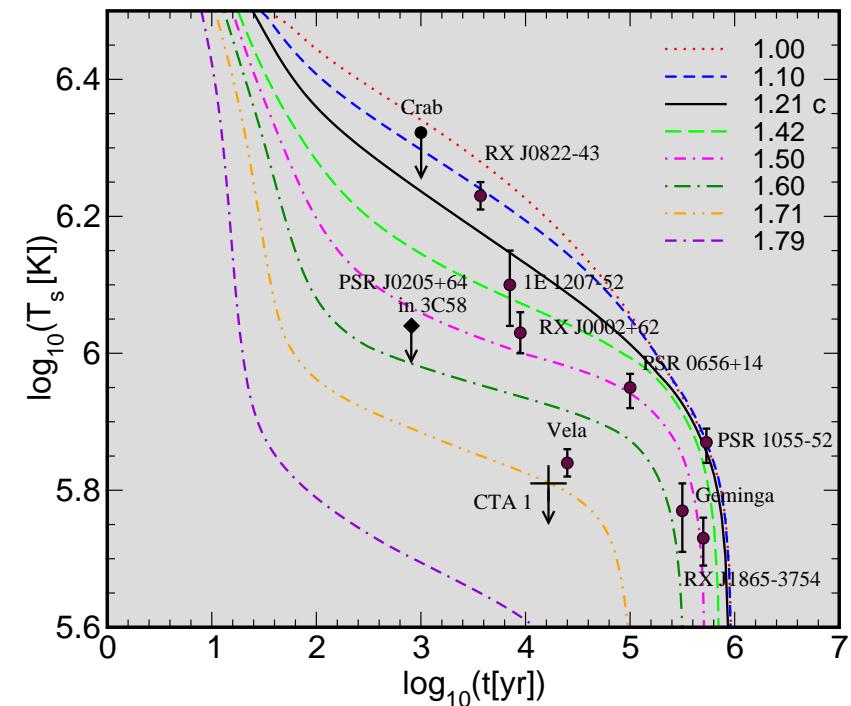
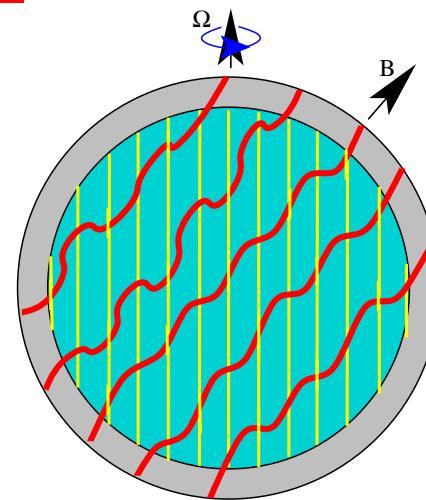
(ii) surface cooling: $t \gtrsim 10^6$ yr

[Alford et al, astro-ph/0411560],

[Blaschke et al, astro-ph/0411619]

Other potential observables

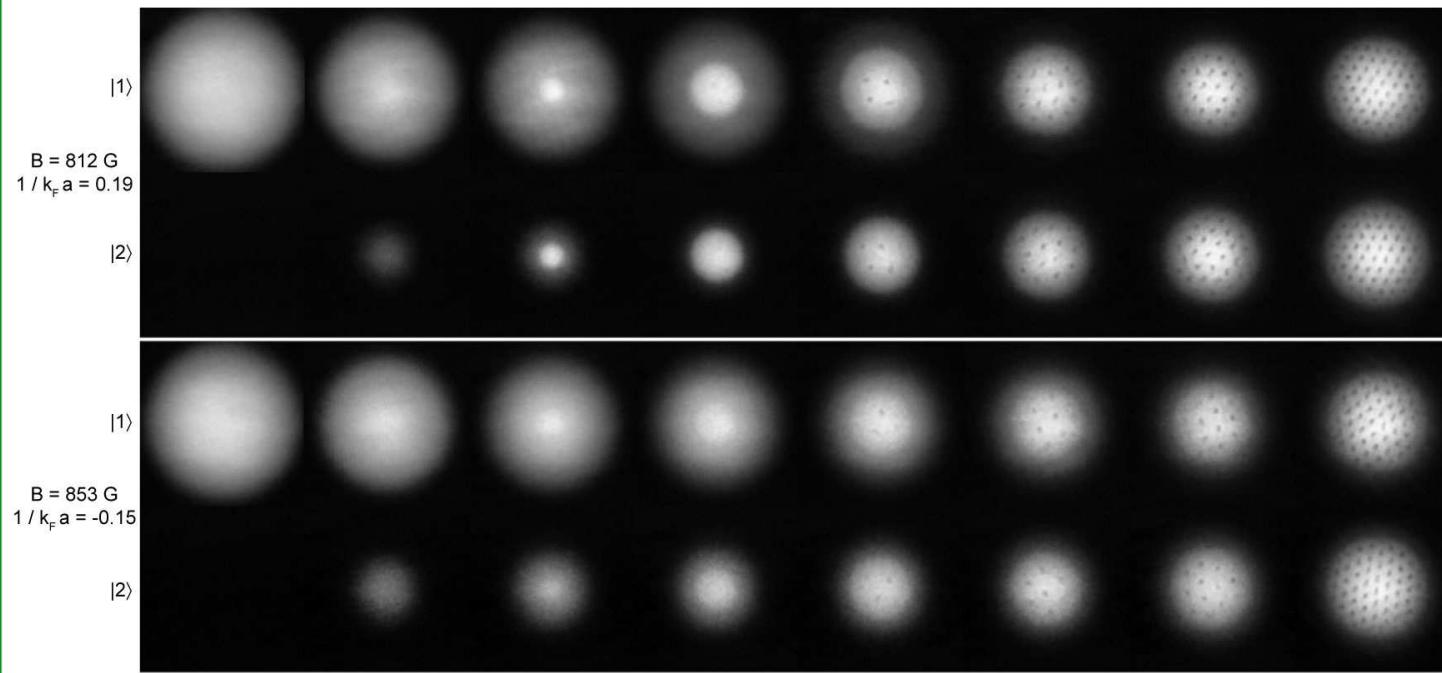
- R-mode instabilities
- Magnetic field decay
- Glitches
- ...



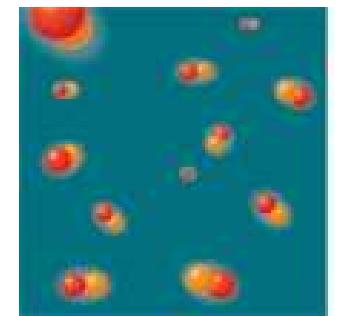
Down to earth ...

☺ Dense quark matter may be “modeled” in a tabletop experiment by studying trapped cold gases of fermionic atoms (e.g., ${}^6\text{Li}$ or ${}^{40}\text{K}$) (\rightarrow talk by Braaten)

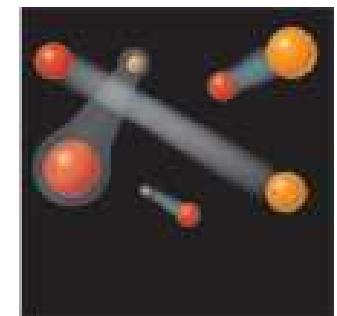
First experimental results:



BEC limit



BCS limit



Zwierlein *et. al.*, cond-mat/0511197*

Partridge *et. al.*, cond-mat/0511752

Summary

- At $\mu \gg \Lambda_{QCD}$, QCD dynamics is weakly coupled, but non-perturbative
- In this limit, QCD can be studied from first principles
- Under conditions in stars, Cooper pairing is unconventional
- There may exist many different phases in the QCD phase diagram as well as in stars
- Physics of stars and physics of matter around us might be closer related than one might naively expect ...

Many problems remain:

- (i) instabilities of gapless phases
- (ii) inhomogeneous ground states
- (iii) search for observables, etc.

